The Mathematical Sublime and Chaos Theory in Kant and Wordsworth

("Pitching Apocalypse")

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Although my remarks today will focus on Kant, Malthus and chaos theory, my starting point is Geoffrey Hartman's reading of Wordsworth, in particular his establishment in *Wordsworth Poetry* of the apostrophe to the Imagination in book 6, and the Ascent of Snowdon in book 13 as “rival high-points” of *The Prelude.* On the one hand, the apostrophe to the Imagination exemplifies "consciousness of self raised to apocalyptic pitch"; on the other hand, Snowdon is Wordsworth’s most astonishing avoidance of apocalypse.” Wordsworth’s *poetic* achievement, exemplified by Snowdon, thus not only depends on but is structured as an evasion of the recognition with which the apostrophe to the Imagination interrupts his narration of crossing the Alps.

I am deeply indebted to Hartman’s work, but this set-up has always troubled me. What follows are steps in an effort to shift what Hartman so memorably identifies as Wordsworth’s “avoidance of apocalypse” into a conceptual landscape where it looks more like a positive accomplishment than an evasion, the discovery of a way of doing something – writing major poetry – without relying on a resource that had been thought necessary to that task – reference to an apocalyptic or anagogical dimension.

Although the advent of the apocalyptic imagination in Hartman's account always involves a “moment of arrest”, that this moment occurs as a result of self-consciousness being “raised” to its highest or apocalyptic “pitch” also locates it in relation to a process, a quickening of subjectivity, which is at once carried to a limit and interrupted, interrupted in so far as the passage to the limit involves a discontinuity, a sudden leap. This quickening can at least partially be described in formal terms (and there is already a hint of mathematical formalization in

Hartman's language, with its echo of "raising to a higher power") as an exponential intensifying feedback loop in which self-consciousness feeds on itself.

I now want to bring this dimension of Hartman's account into alignment with a feature of Kant's analysis of the mathematical sublime which tends to be overlooked: his recourse to something like patterns of geometrical or exponential expansion in the staging of the experience of the mathematical sublime.

Most accounts of Kant on the mathematical sublime, from Monk to Hertz to Ferguson, portray it as precipitated out of the imagination's struggle and failure to comprehend boundless multiplicity in an intuition, or in Hertz's words "to bring a long series or a vast scattering under some sort of conceptual unity."

Consider, however, this passage from the closing paragraph of section 26 of the Analytic of the Sublime:

> "Examples of the mathematically sublime of nature in mere intuition are provided by all those cases in which we are given, not so much a larger numerical concept as rather a greater unity as measure (for shortening the numerical series) for the imagination. A tree that we estimate by the height of a man may serve as a standard for a mountain, and, if the latter were, say, a mile high, it could serve as the unit for the number that expresses the diameter of the earth, in order to make the latter intuitable; the diameter of the earth could serve as the unit for the planetary system so far as known to us, this for the Milky Way, and the immeasurable multitude of such Milky Way systems, called nebulae, which presumably constitute such a system among themselves in turn, does not allow us to expect any limits here. Now in the aesthetic judging of such an immeasurable

> whole, the sublime does not lie as much in the magnitude of the number as in the fact that as we progress we always arrive at ever greater units.

Following Kant, we may ask what drives ("treibt") this process as it moves up from the earth-bound human figure to the boundlessness of the heavens. For there is nothing in the goal of measurement *per se*, "that would necessitate pushing the magnitude of the measure…to the boundaries of the faculty of the imagination."

A different goal, a different finality, must thus be conceived as driving this process to reproduce itself *ad infinitum* in thought. An answer is hinted at by the fact that Kant's illustrations of such a "progressive, growing" series begin with reference to the human body as a standard of measure. The introduction of the human body points to the involvement of a self-reflexive operation: the subject taking its own body, rather than an object external to it, as a unit of measure. The self-reflection is also a self-magnification since its result is an enhancement of the subject's cognitive mastery of its environment. From this point of view, a different "progressively growing series" emerges in which the securing of each new unit of measurement subserves the recursive production of ever greater levels of cognitive power. The projection of progressively greater units of measure – a body, a tree, a mountain, etc. is simultaneously a projection of progressively greater objects *and* of a progressively empowered theoretical subject. Kant famously concludes that "…it is not the object of sense, but the use which the Judgement naturally makes of certain objects on behalf of this latter feeling, that is absolutely great; and in comparison every other use is small." But this leap towards an absolutely great "*use* which the Judgment makes of objects…in comparison with which every other use is small…" - namely to prefigure the transcendence of the imagination by the ideas of reason, springs forth from the structure of self-magnification we have been tracing.

It may seem a long way from Kant's 1790 *Critique of Judgment* and its account of the mathematical sublime to Malthus's 1798 *Essay on the Principle of Population.* But it is not entirely coincidental that Malthus should base his argument, not on the flaws of fallen human nature, but on the claim that "Population, when unchecked, increases in a geometrical ratio…," while "Subsistence increases only in an arithmetical ratio." The necessary corollaries of this disparity, the means by which the "superior power of population is repressed, and the actual population kept equal to the means of subsistence" are "misery and vice."

Compared with the feelings of spiritual elevation Kant's analysis is meant to inspire in explaining, Malthus's affirmations are a down, but they luxuriate in their own grim calculus; and, no less than Kant's resolution of the conflict between "a progressively growing series" and the limits of the imagination, present themselves as expressions of "reason's demand for totality," that is, for a comprehensive representation that deduces from the divergence, when taken in isolation, of two tendencies, the operation and consequences of a regulative principle, denominated "reason" in Kant and "the strong law of necessity" in Malthus.

In this connection, Malthus's queer derivation of "vice" as a necessary consequence of the inherent inequality of "the two powers" is particularly striking for its use of a pattern of argumentation akin to that by which Kant derives the phenomenon of the mathematical sublime. You have the passage in its entirety. I cite only the sentences I have italicized:

> *Impelled to the increase of his species by an equally powerful instinct, reason interrupts his career and asks him whether he may not bring beings into the world for whom he cannot provide the means of subsistence…And this restraint almost necessarily, though not absolutely so, produces vice.*

Although Malthus avoids being explicit, the clear implication is that reason leads man (and "man" it clearly is) to dissipate his sexual energies outside the conjugal bond so as not to compromise his physical and social autonomy. The ideological cast of such "reasoning" scarcely requires comment. That said, the structural congruence with Kant's epistemo-aesthetic scenario of Malthus's claim that "reason interrupts…" man in the "career" of his instinct to be fruitful and multiply is remarkable and bespeaks a shared imaginative preoccupation.

In 1838, Pierre Francois Verhulst published a brief "Notice sur la loi que la population suit dans son accroissement," followed by more extended papers in 1845 and 1847. Verhulst's intention was not to offer a critique of Malthus' analysis, but rather to supplement it. Malthus's mathematical formalizations basically do not proceed beyond his two fundamental axioms about the exponential growth of population and the linear growth of the means of subsistence. To represent the interaction of
"these two different ratios," he turns, as we have seen, to the language of "misery" and "vice" rather than to a mathematical formula. A professor of Mathematics with an interest in social statistics, Verhulst was drawn to formulate a more complex "law of population" which would mathematically integrate that interaction – that is, model mathematically the dynamics of the system formed by the relationship between a population and its environment.

To accomplish this, Verhulst introduced a new postulate and corollary. The postulate was that, for a given environment and a given population, there is a *maximum* possible population which the environment can support. The corollary was that the nearer a population approaches to that maximum, the greater the resistance to its growth and thus the slower its rate of growth. Thus the growth of a population that was at some distance from the maximum supported by its environment might well advance according to Malthus's geometric "power of population" while the rate of growth of the same population if it were near the maximum would be proportionately slowed.

Mathematically, the problem for Verhulst was to devise a factor that would have a variable effect on the rate of growth of a population depending on its size at a given time relative to its potential maximum size. His solution was, first to designate the maximum population simply as 1, so that the size of varies between 0 and 1 rather than pressing towards infinite expansion, and second to define the constraining factor as the difference between the maximum population and the actual population at a given time. The evolution over time of a given population in a given environment could thus be represented by the recursive equation $p_{n+1} = R*p_n(1- p_n)$, where R is the growth rate, p_n is the population at a given time and p_{n+1} is the new population that is the product of the interaction of these factors.

However abstract this formula may be as a model of the actual dynamics of population growth and decay (and Verhulst acknowledges as much), it has shown its applicability in a number of areas, and in particular, beginning in the 1970s, as a paradigmatic illustration of the potential for chaotic behavior in a simple system.

This was hardly a result anticipated by Verhulst, but its disclosure reveals, I would suggest, a significant displacement and dissemination of the moment of disjunction which is central to Kant's analysis of the mathematical sublime and correlatively to Hartman's notion of "consciousness of self raised to apocalyptic pitch." On the one hand, the self-reflexive operations determined by Verhulst's model generate sequences whose trajectories, per definition, remains within a bounded space rather than "going to infinity." On the other hand, that these orbits may be chaotic means that, while they may revisit a "neighborhood" - you may think here of Wordsworth's "spots of time" - , they may never return to the same point and thus be open-ended in this regard. The finite is here opposed, not by an infinite which transcends it, but by an indefiniteness which is immanent to it. To borrow from Paul de Man writing about excuses in Rousseau, an "isolated textual event...is disseminated throughout the entire text." The event is textual if we accept that systems of numerical representation are texts, a conclusion which may be the burden of Kant's analytic of the sublime.

(handout for "The Mathematical Sublime and Chaos Theory in Kant and Wordsworth" - Wilner)

1. Kant, *The Critique of Judgment,* Chapter 35, Section 26, para. 13

"Examples of the mathematically sublime in nature in mere intuition are provided for us by all those cases where what is given to us is not so much a greater numerical concept as rather a great unity as measure (for shortening the numerical series) for the imagination. A tree that we estimate by the height of a man may serve as a standard for a mountain, and, if the latter were, say, a mile high, it could serve as the unit for the number that expresses the diameter of the earth, in order to make the latter intuitable; the diameter of the earth could serve as the unit for the planetary system so far as known to us, this for the Milky Way, and the immeasurable multitude of such Milky Way systems, called nebulae, which presumably constitute such a system among themselves in turn, does not allow us to expect any limits here. Now in the aesthetic judging of such an immeasurable whole, the sublime does not lie as much in the magnitude of the number as in the fact that as we progress we always arrive at ever greater units; the systematic division of the structure of the world contributes to this, representing to us all that is great in nature as in its turn small, but actually representing our imagination in all its boundlessness, and with it nature, as paling into insignificance beside the ideas of reason if it is supposed to provide a presentation adequate to them."

(trans. Paul Guyer and Eric Matthews)

2. Malthus, *An Essay on the Principle of Population*, Chapter 2

No limits whatever are placed to the productions of the earth; they may increase for ever and be greater than any assignable quantity. Yet still the power of population being a power of a superior order, the increase of the human species can only be kept commensurate to the increase of the means of subsistence by the constant operation of the strong law of necessity acting as a check upon the greater power. The effects of this check remain now to be considered.

Among plants and animals the view of the subject is simple. They are all impelled by a powerful instinct to the increase of their species, and this instinct is interrupted by no reasoning or doubts about providing for their offspring. Wherever therefore there is liberty, the power of increase is exerted, and the superabundant effects are repressed afterwards by want of room and nourishment, which is common to animals and plants, and among animals by becoming the prey of others.

The effects of this check on man are more complicated. ***Impelled to the increase of his species by an equally powerful instinct, reason interrupts his career and asks him whether he may not bring beings into the world for whom he cannot provide the means of subsistence.*** In a state of equality, this would be the simple question. In the present state of society, other considerations occur. Will he not lower his rank in life? Will he not subject himself to greater difficulties than he at present feels? Will he not be obliged to labour harder? and if he has a large family, will his utmost exertions enable him to support them? May he not see his offspring in rags and misery, and clamouring for bread that he cannot give them? And may he not be reduced to the grating necessity of forfeiting his independence, and of being obliged to the sparing hand of charity for support?

These considerations are calculated to prevent, and certainly do prevent, a very great number in all civilized nations from pursuing the dictate of nature in an early attachment to one woman. ***And this restraint almost necessarily, though not absolutely so, produces vice.*** Yet in all societies, even those that are most vicious, the tendency to a virtuous attachment is so strong that there is a constant effort towards an increase of population. This constant effort as constantly tends to subject the lower classes of the society to distress and to prevent any great permanent amelioration of their condition

(my emphasis)

3. Verhulst's "logistic equation" (cast as a difference equation):

$$p_{n+1} = r \cdot p_n(1-p_n)$$

p_n = ratio of actual population at time "n" to the maximum possible population

r = birth rate

Malthus's exponential "power" or "law of population" cast in the same form would be:

$$p_{n+1} = r \cdot p_n$$

where p_n is the population at time "n."